7.1 continued

24. $\int_0^1 (x^2+1)e^{-x}\,dx = -e^{-x}(x^2+1)\Big|_0^1 + 2\int_0^1 xe^{-x}\,dx$, $\quad u = x^2+1 \quad dv = e^{-x}dx$, $\quad du = 2x\,dx \quad v = -e^{-x}$

$= \left(1 - \frac{2}{e}\right) + 2\left[-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right]$

$= \left(1-\frac{2}{e}\right) + 2\left(0 - \frac{1}{e}\right) + 2\left(1 - \frac{1}{e}\right) = \boxed{3 - \frac{6}{e}}$

26. $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy = 2\sqrt{y}\ln y\Big|_4^9 - 2\int_4^9 \frac{1}{\sqrt{y}}\,dy$, $\quad u = \ln y \quad dv = y^{-1/2}dy$, $\quad du = \frac{1}{y}dy \quad v = 2\sqrt{y}$

$= (6\ln 9 - 4\ln 4) - 4\sqrt{y}\Big|_4^9$

$= \boxed{6\ln 9 - 4\ln 4 - 4}$

40. $\int_0^\pi e^{\cos t}\sin 2t\,dt = \int_0^\pi e^{\cos t}(2\sin t\cos t)\,dt$ by a trig identity

Now, use the substitution $w = \cos t$, $dw = -\sin t\,dt$, $-dw = \sin t\,dt$

t	w
0	1
π	-1

$2\int_0^\pi e^{\cos t}\cos t(\sin t\,dt) = -2\int_1^{-1} e^w\cdot w\,dw = 2\int_{-1}^1 we^w\,dw$ after

using the identity $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

Now, $2\int_{-1}^1 we^w\,dw = 2\left[we^w\Big|_{-1}^1 - \int_{-1}^1 e^w\,dw\right]$, $\quad u = w \quad dv = e^w dw$, $\quad du = dw \quad v = e^w$

$= 2\left[\left(e + \frac{1}{e}\right) - \left(e - \frac{1}{e}\right)\right] = \boxed{\frac{4}{e}}$

42. $\int \sin(\ln x)\,dx = \int \sin u(e^u\,du)$ after using the substitution

$u = \ln x$
$du = \frac{1}{x}dx$
$x\,du = dx$
$e^u du = dx$

$= \int e^u \sin u\,du$

$= \frac{1}{2}e^u(\sin u - \cos u) + C$

where I've used the fact that we did this exact problem in class

$= \boxed{\frac{1}{2}x(\sin(\ln x) - \cos(\ln x)) + C}$

7.1 Solutions — Math 31

2. $\int \theta\cos\theta\,d\theta = \theta\sin\theta - \int \sin\theta\,d\theta$ $\qquad u=\theta \quad dv=\cos\theta\,d\theta$
$= \boxed{\theta\sin\theta + \cos\theta + C}$ $\qquad du = d\theta \quad v = \sin\theta$

8. $\int t^2\sin\beta t\,dt = -\frac{1}{\beta}t^2\cos\beta t + \int \frac{2}{\beta}t\cos\beta t$, $\quad u=t^2 \quad dv=\sin\beta t\,dt$
$du = 2t\,dt \quad v = -\frac{1}{\beta}\cos\beta t$
$= -\frac{1}{\beta}t^2\cos\beta t + \frac{2}{\beta}\left[\frac{1}{\beta}t\sin\beta t - \frac{1}{\beta}\int\sin\beta t\,dt\right]$, $\quad u=t \quad dv=\cos\beta t\,dt$
$= \boxed{-\frac{1}{\beta}t^2\cos\beta t + \frac{2}{\beta^2}t\sin\beta t + \frac{2}{\beta^3}\cos\beta t + C}$ $\qquad du=dt \quad v=\frac{1}{\beta}\sin\beta t$

10. $\int \sin^{-1}x\,dx = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$, $\quad u=\sin^{-1}x \quad dv=dx$
$du = \frac{1}{\sqrt{1-x^2}}dx \quad v = x$
$= x\sin^{-1}x + \frac{1}{2}\int\frac{dw}{\sqrt{w}}$, $\quad w=1-x^2$
$dw = -2x\,dx$
$-\frac{1}{2}dw = x\,dx$
$= x\sin^{-1}x + \sqrt{w} + C = \boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$

12. $\int p^5\ln p\,dp = \frac{1}{6}p^6\ln p - \frac{1}{6}\int p^5\,dp$, $\quad u=\ln p \quad dv=p^5\,dp$
$= \boxed{\frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C}$ $\qquad du=\frac{1}{p}dt \quad v=\frac{1}{6}p^6$

18. $\int e^{-\theta}\cos 2\theta\,d\theta = -e^{-\theta}\cos 2\theta - 2\int e^{-\theta}\sin 2\theta\,d\theta$, $\quad u=\cos 2\theta \quad dv=e^{-\theta}d\theta$
$du=-2\sin 2\theta\,d\theta \quad v=-e^{-\theta}$
$= -e^{-\theta}\cos 2\theta - 2\left[-e^{-\theta}\sin 2\theta + \int 2e^{-\theta}\cos 2\theta\,d\theta\right]$, $\quad u=\sin 2\theta \quad dv=e^{-\theta}d\theta$
$du=2\cos 2\theta\,d\theta \quad v=-e^{-\theta}$
$= -e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta - 4\int e^{-\theta}\cos 2\theta\,d\theta$. After Some Algebra

$$\int e^{-\theta}\cos 2\theta\,d\theta = \frac{e^{-\theta}(2\sin 2\theta - \cos 2\theta)}{5} + C$$